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A refined physical model of the clarinet using a variable air jet height.

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Abstract—A time-domain formulation of a lumped model approximation of a clarinet reed excitation mechanism is presented. The lumped model is based on an analytical representation of the flow within the reed channel, incorporating a contraction coefficient (vena contracta factor) that is defined as the ratio of the effective flow over the Bernoulli flow. This coefficient has been considered to be constant in previous studies focusing on sound synthesis. In this paper it will be treated as a function of the reed opening, varying between 0 and 1 as predicted by boundary layer flow theory. Focussing on a specific mouthpiece geometry, the effect of modelling a variable air jet height on the synthesised sound is analysed.

I. INTRODUCTION

Modelling single-reed woodwind instruments using a time-domain approach has been introduced by Schumacher [1] and was then extended and built upon by several authors. Time-domain calculations can deal with non-linear oscillations and are able to model both the transient and the steady state behaviour of the system. The oscillation of the reed is mostly simulated using a one-mass (lumped) model. Originally formulated as a linear model [1], [2], [3], [4], subsequent studies introduced models with non-constant parameters [5] and methods for estimating parameters from distributed models of the reed [6] such that the vibrational behaviour of the lumped model is similar to that of the distributed model. In the present paper an approach similar to [6] has been carried out, but in this case the parameters have been estimated by a two-dimensional model for the reed [7]. Thus it has been possible to inherently take into account any torsional modes of the reed, that have been proved to affect the sound quality [8].

Concerning the fluid dynamics of the model, the formulation of the flow inside the mouthpiece was, in most cases, quite moderate using either a Bernoulli flow occupying the whole reed channel [3], [6], [9], [10] or considering the formation of an air jet in the reed channel with a constant height [4], [11]. In this paper a more analytical formulation for the flow in the reed channel is adopted, using a variable air jet height predicted by Boundary Layer Flow theory. This theory is presented in the next section, followed by the numerical formulation of the lumped model and the results obtained from the simulations.

II. FLOW IN THE REED CHANNEL

The Euler equations for inviscid, incompressible flow can be used to calculate the Bernoulli equation [12]

$$\frac{1}{2}\rho|u|^2 + p = \text{const.} \quad (1)$$

However, the viscosity near the walls cannot be neglected for the flow through the reed channel. Frictional forces tend to retard the flow in a layer of thickness b next to the wall, which is defined as the boundary layer. This results in the formation of an air jet inside the reed channel. The height of this air jet can be described using a non-dimensional parameter α , the vena contracta factor, defined as $\alpha = u/u_B$, where u_B is the Bernoulli flow and u the effective flow in the channel. If, furthermore, the height of the channel is taken to be h then

$$\alpha = 1 - \frac{b}{h}. \quad (2)$$

This parameter α is very difficult to determine experimentally or to estimate numerically. Most of the earlier studies on single reed instruments either neglect it or consider it to be constant. It is claimed though [13] that in order to improve the sound produced by physical modelling, the aero-acoustics of the instrument have to be studied thoroughly. The theoretical description of α by van Zon [14], also adopted in the present paper, seems to match the experiments he carried out with a 5% error. The usefulness and reliability of this formulation have been questioned [13], because the whole study was based on a quasi-static approach i.e. assuming that the flow in the mouthpiece with an oscillating reed would be equal to that of a static reed with the same displacement at each instant. Hence the experiments verifying its validity were confined to a static nature. In 2007, using a Lattice-Boltzman method, da Silva *et al.* [15] simulated the flow through the reed channel using a moving boundary for the oscillating reed. The dynamical data obtained for the vena contracta factor displays a qualitative resemblance to van Zon's theory, which suggests that using a theory-based variable- α formulation might provide a useful refinement of the lumped reed model. An additional reason for the current authors to use such a refined model is that they intend to employ it to inverse modelling of the clarinet reed system, i.e. estimating its parameters from experimental data. In that scenario it seems more appropriate to pre-assume that α

is non-constant. In addition, the effects of turbulence induce further uncertainties, especially on the exact location of the point where the flow reattaches to the walls due to the Coanda effect [16]. Furthermore, the effect of the lateral slits to the total flow has not yet been fully examined. So, even though using a two-dimensional reed model allows the analytical calculation of the side openings, their exact influence on the total flow remains unknown. What has been done was an effort to match the experimental results obtained by Valkering [17] by scaling down the total opening surface.

Now, limiting our study to long reed channels, which is the case for the specific geometry of the mouthpiece used [7], an air jet is formed at the entrance of the reed channel. This air jet is expected to reattach to the wall after a distance x_w (of the same order with h) from the entrance. Assuming a linear flow-velocity profile within the boundary layer, integration of the von Karman equation gives [18]

$$\frac{(L - x_w)}{h} = \frac{1}{6} Re \left(4 \frac{b}{h} + 9 \ln \left(1 - \frac{b}{h} \right) + 5 \frac{b}{b-h} \right) \quad (3)$$

where $L = 0.0014\text{m}$ is the length of the reed channel and Re the Reynolds number. The geometry of the reed channel as well as the formation of the boundary layer and the reattachment to the wall are depicted in Figure 1.

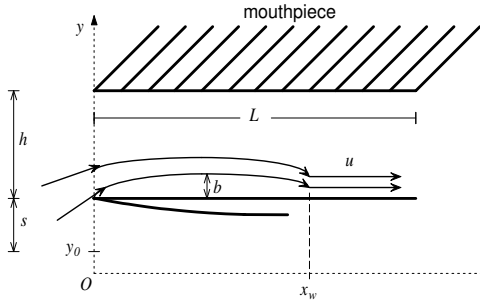


Fig. 1. Geometrical parameters for the flow in the reed channel during the reed motion, where y_0 denotes the equilibrium point of the reed.

This equation can be solved numerically for b . However, there is a critical value for b , (that can be calculated from the conservation of mass and momentum [18]) namely $b_c = 0.2688 h$, above which transition to a fully developed Poiseuille flow occurs at a distance L_1 from the channel entrance, resulting in [17]

$$\alpha = \frac{1}{\sqrt{\frac{1}{(1-b_c)^2} + 24 \frac{L-L_1}{Re h}}} \quad (4)$$

III. NUMERICAL FORMULATION

It can be argued that keeping the effective mass and damping of the reed constant in a lumped model formulation, still captures most of the dynamics of the system, at least for small-amplitude oscillations [6]. Under this assumption, the equation of motion becomes

$$m \frac{d^2 y_L}{dt^2} + g \frac{dy_L}{dt} + K_\alpha (\Delta P) (y_L - y_0) = \Delta P \quad (5)$$

where y_L is the middle point of the free edge of the reed, m the mass per unit area and g the damping per unit area. The effective stiffness per unit area, K_α , and the reed effective surface S_r , have been estimated using a distributed two-dimensional model for the reed-mouthpiece-lip system [7]. The flow inside the reed channel is given by $u_f = \alpha u_B$ and the flow induced by the oscillation of the reed is

$$u_r = \frac{ds}{dt} S_r \quad (6)$$

with s the displacement of the reed tip from its equilibrium point. The mouthpiece pressure p can be decomposed into a wave going into (p^+) and out (p^-) of the bore, which are related to the total volume flow $u = u_r + u_f$ by

$$Z_0 u = p^+ - p^-, \quad (7)$$

where Z_0 is the characteristic impedance at the mouthpiece entry.

Combining equations (1) and (7) yields the non-linear equation for u_f

$$\text{sign}(u_f) \frac{\rho}{2S_f^2} u_f^2 + Z_0 u_f + (2p^- - p_m + Z_0 u_r) = 0 \quad (8)$$

where p_m is the blowing pressure. The vena contracta factor α is included into equation (8) as a scaling factor of the opening surface S_f . The in-going pressure can be computed from the total flow u and then effectively convolved with the reflectance of the aircolumn, as calculated in [19] using the wave digital modelling method, which completes the feedback loop. Equation (5) is solved using the impulse invariant method, as described in [6] and equation (8) can be solved using Newton's method, with the previous value of u_f as an initial guess.

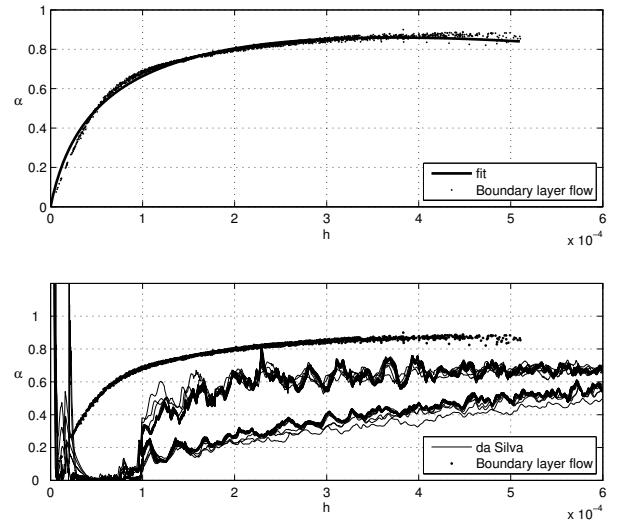


Fig. 2. α as a function of h (top) and a comparison of α predicted by boundary layer flow and a Lattice-Boltzmann simulation (bottom).

In order not to have to solve equation (3) for b at every time step and then compare its value with b_c , the following procedure was used. The simulation was run several times with different blowing pressures in order to get the reed oscillating at all amplitudes and the values for the vena contracta factor have been stored (assuming that $x_w \approx 2h$). Then a least squares fit was performed on the obtained data (using QR decomposition to solve the badly conditioned normal equations) giving α as a function of the reed opening h . The result of this was

$$\alpha(h) = 0.39 \ln(h + 16 \cdot 10^{-6}) - 993.92 \sin(h) + 4.27. \quad (9)$$

This function has been used throughout the simulations to compute α given the reed opening, and the goodness of this fit (also confirmed statistically using a *gamma* distribution) can be seen at the top of Figure 2. At the bottom of the Figure the data obtained by the quasi-static formulation is compared with that of the dynamical simulation carried out in [15]. Even though the values for α are not the same, they exhibit a similar trend. The two separate branches of the dynamical data can be explained by the fact that a dynamic model, unlike a static model, can distinguish between the opening and closing motion of the reed. Both approaches point towards a non-constant formulation for α .

IV. RESULTS

The lumped model was excited using a blowing pressure that rapidly increased from 0 to p_1 and then slowly and linearly increased from p_1 to p_2 for the duration of the sound until it quickly faded out at the end. The obtained results for the pressure in the mouthpiece are compared for the cases of a constant and a varying α . For the case of a constant α , the value chosen was $\alpha = 0.86$, calibrated in such a way as for the pressure in the mouthpiece to have the same amplitude during the steady state with that of the case of α varying as a function of h .

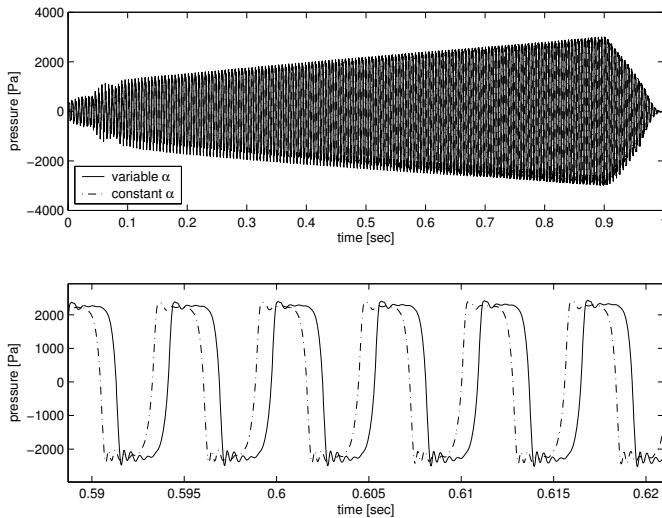


Fig. 3. Mouthpiece pressure with a constant and variable α (top) and a zoom-in to observe the “shift” (bottom).

At the steady state of the sound the difference of the two methods is only confined to a slight pitch bending, almost inaudible, but visible by zooming in the pressure plot (see Figure 3).

However, during the transient behaviour, the difference between the two cases becomes more significant. Using a constant value for α introduces audible low frequency components caused by some form of autonomous reed oscillation. An interesting observation is that these frequencies do not match the reeds resonance frequency, but are visible around half of this value. Such a phenomenon has been attributed to period doubling [20] for free organ reeds. However, in this case it may be described best as reed inertia effects. The phase space of the pressure signal during part of the transient, depicted in Figure 4, shows the effect of inertia forces to the system. The two small circles observed at the peaks of the motion can be responsible for the increase of the oscillation period, giving similar results to those of period doubling. This can be compared with the phase space at the steady state (see Figure 5), where this effect becomes negligible, as the bore natural frequencies become dominant. On the other hand, using a variable α causes the reed induced frequencies to become less significant and almost inaudible. The spectrograms of the two signals can be compared in Figure 6.

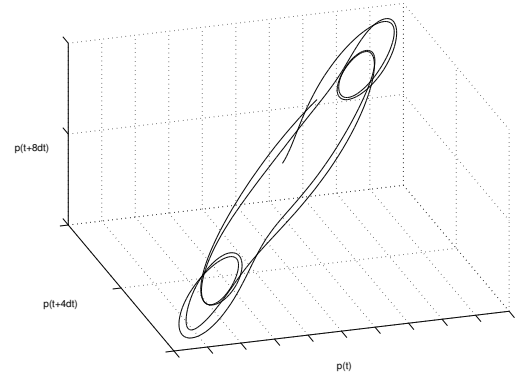


Fig. 4. Phase space trajectory of the pressure signal during the transient, in the case of constant α .

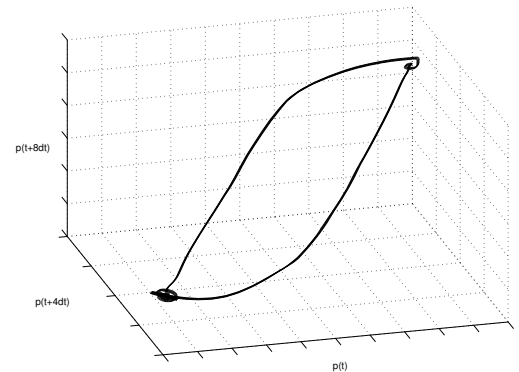


Fig. 5. Phase space trajectory of the pressure signal at the steady state.

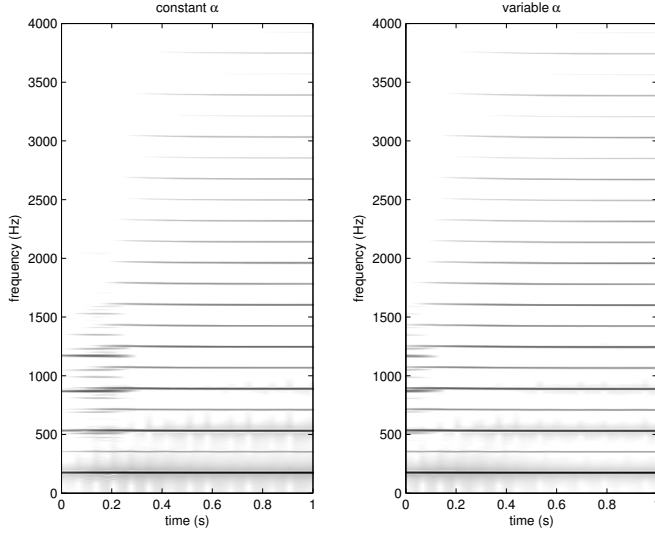


Fig. 6. Spectrograms of the mouthpiece pressure computed with $p_1 = 1100$, $p_2 = 2000$ and a 44.1 kHz sample rate.

Furthermore, for a signal produced with a constant blowing pressure, different values of a constant α have been used, in order to compare its influence on the frequency spectrum. The differences between the odd harmonics are not easily seen in Figure 7, so the results for the first 5 odd harmonics are stated in Table I. It can be observed that there is a strong influence of the variation of α on the third harmonic. The first harmonic remains almost constant, whereas the fifth seems to be changing with no proportionality to α . For the higher harmonics we revert to a proportional behaviour especially for low values of α .

Another difference when using a variable air jet height instead of a constant one can be observed on the threshold blowing pressure needed to get the reed oscillating. Using a constant α , this pressure was around 1070 Pa, whereas in the case of varying the height of the air jet the threshold blowing pressure was reduced to 1015 Pa.

V. CONCLUSION

A method has been proposed to incorporate an analytical flow model into a lumped model approximation of the reed-mouthpiece system. It has been shown that the resulting sound is different compared to the sound produced by the previously

TABLE I
MAGNITUDE (IN dB SOUND PRESSURE LEVEL) OF THE FIRST FIVE ODD HARMONICS FOR DIFFERENT VALUES OF α .

harmonic	$\alpha = 0.58$	$\alpha = 0.72$	$\alpha = 0.86$	$\alpha = 1$
1	64.62	65.10	64.71	64.64
3	49.40	51.29	52.40	53.31
5	44.34	43.94	46.56	46.07
7	37.86	40.23	40.45	43.04
9	29.30	34.56	36.91	36.92

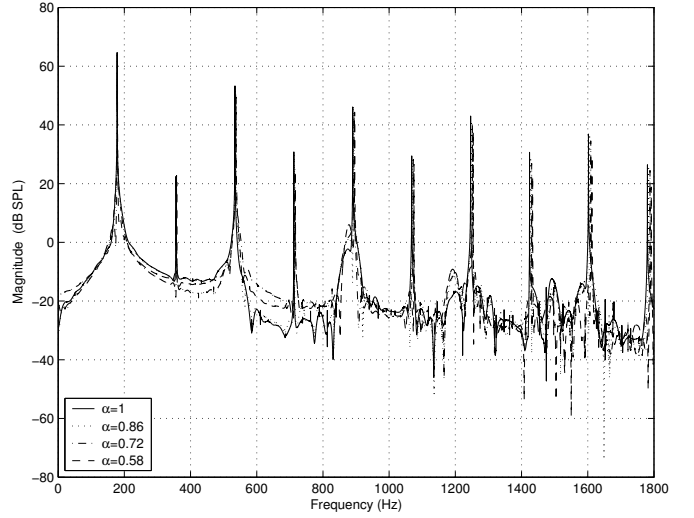


Fig. 7. Frequency spectrum for different values of the air jet height for a ten second long signal.

used flow models. In particular, there is a significant difference during the transient. A small variation can also be observed at the steady state of the sound.

Unfortunately, concerning modelling the flow in the reed channel, there are no known dynamical experimental results that confirm the variable- α formulation. Indeed, the motivation for including a variable air jet height originates from the authors' intention to use the lumped model in an inverse-modelling procedure based on data obtained under real playing conditions. Such data are expected to behave in a more complex way than that predicted by a simple model with a constant α . Hence the variable- α formulation is a suitable starting point for developing the inverse approach.

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